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FLUID FLOW IN AN ASYMMETRIC CHANNEL OF VARIABLE CROSS-SECTION WITH SLIP CONDITION AT THE WALL

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ABSTRACT

This study deals with the effects of slip and phase difference in a steady flow of an incompressible asymmetric rigid channel with permeable walls. It is assumed that the effect of fluid absorption through permeable walls is accounted by prescribing flux as a function of axial distance. The perturbation method is applied to linearize the non-linear governing equations by assuming the ratio of inlet width to wavelength to be small. Effects of the above parameters on the velocity profile, mean pressure drop and wall shear stress are studied in detail and explained graphically.

Keywords: Permeable channel, slip parameter, asymmetry.

1 INTRODUCTION

The study of the flow of viscous fluid in an asymmetric channel of varying cross section with permeable walls is much interested in recent years in view of its numerous applications in many physiological and engineering problems. Fluid flow in renal tubules was studied by many authors. Mathematical modeling of the flow in proximal renal tubule was first studied by Macey [16] where he considered the flow of an incompressible viscous fluid through a circular tube with a linear rate of reabsorption at the wall. Bulk flow in the proximal tubule decays exponentially with the axial distance was calculated by Kelman [5]. Then, Macey [17] used this condition to solve the equations of motion to find the average pressure drop. Marshall *et.al*[7] and Palatt *et al*[12] studied the physical conditions existing at the rigid permeable wall instead of prescribing the flux /radial velocity at the wall.

In all the above studies the researcher considered the renal tubule to be symmetry. But in general, renal tubules may not be symmetric throughout their length. A hydrodynamical aspect of an incompressible viscous fluid in a circular tube of varying cross-section with reabsorption at the wall is studied by Radhakrishnamacharya *et al*[14]. Flow in rigid tubes of slowly varying cross-section with absorbing wall is studied by Peeyush Chandra and Krishna Prasad [13]. Fluid flow through a diverging/converging tube with variable wall permeability was studied by Chaturani and Ranganatha [2].

The concept of slowly varying flow is given by Manton [6] where he obtained an asymptotic series solution for the low Reynolds number flow through an axisymmetric tube, where radius varies slowly in the axial direction.

The effects of slope parameter and reabsorption coefficient on the flow of fluid in a symmetric channel with varying cross section with no-slip velocity at the walls are studied by Muthu and Tesfahun [9].

In all the above studies the researchers have taken the boundary condition at the wall to be a no-slip condition, whereas the no-slip condition is one of the aspects on which the mechanics of the viscous liquids is built. However, there are many situations where this assumption does not hold [15]. Elshahed [8] illustrated the significance of the effect of slip at the wall. Also, the slip would be most useful for certain problems in chemical engineering and other applications ([15],[3],[4],[18],[19]). Fluid flow through the non-uniform channel with permeable wall and slip effect in symmetry channel is studied by [11]. Further, Muthu and Tesfahun [10] discussed the flow through in renal tubule by considering the asymmetric channel of varying cross-section, whereas Waseem et al [20] study the effect of slip on fluid flow in a channel of the slowly varying cross section.

Thus, in this paper, an attempt is made to understand the flow through renal tubule of asymmetry channel of varying cross-section and a slip velocity at the walls of the channel.

2 MATHEMATICAL FORMULATION

Here we consider an incompressible fluid flow through the asymmetric channel with a slowly varying cross-section. The boundaries of the channel wall are taken by Muthu *et al*[10]as

$$\left.\begin{aligned} \eta_1(x) &= d_1 + a_1 \cos\left(\frac{2\pi x}{\lambda}\right) \quad \text{..... upper wall} \\ \eta_2(x) &= -d_2 - b_1 \cos\left(\frac{2\pi x}{\lambda}+\phi\right) \quad \text{..... lower wall} \end{aligned}\right\}, \text{ where } \quad 0 \le x \le \lambda \tag{1}$$

Where d_1 and d_2 are the half width of the channel from the x-axis to $\eta_1(x)$ and $\eta_2(x)$ respectively at the inlet (at x=0), a_1 and b_1 are amplitudes and λ is the wavelength further a_1 , b_1, d_1 , d_2 , ϕ satisfies the condition

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos(\phi) \le \left(d_1 + d_2\right)^2 \tag{2}$$

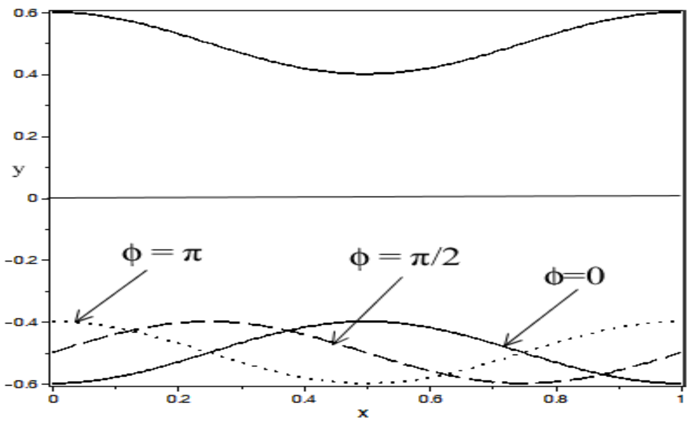


Figure 1. Geometry of the channel

We shall consider the motion of the fluid to be laminar and steady and the channel to be long enough to neglect the initial and end effects. The equations of continuity and momentum are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{4}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{5}$$

Where u and v are the velocity components along the x and y axes respectively, p is the pressure, ρ is the density of the fluid and $\nu=\dfrac{\mu}{\rho}$ is kinematic viscosity.

In order to complete the formulation of the problem, the boundary conditions are taken as follows.

(a) The tangential velocity at the wall is not zero. That is,

$$u+\frac{d\eta_1}{dx}v=-\frac{\sqrt{\gamma}}{\beta}\left(\frac{\partial u}{\partial y}+\frac{d\eta_1}{dx}\frac{\partial v}{\partial y}\right) \quad \text{at} \quad y=\eta_1(x) \tag{6}$$

$$u+\frac{d\eta_2}{dx}v=-\frac{\sqrt{\gamma}}{\beta}\left(\frac{\partial u}{\partial y}+\frac{d\eta_2}{dx}\frac{\partial v}{\partial y}\right) \quad \text{at} \quad y=\eta_2(x) \tag{7}$$

Where β is slip parameter and γ is the specific permeability of the porous medium.

(b) The reabsorption has been accounted for by considering the bulk flow as a decreasing function of *x*. That is, the flux across a cross-section is given by

$$Q(x)=\int_{\eta_2(x)}^{\eta_1(x)} u(x,y)\,dy=Q_o\,F(\alpha x) \quad , \tag{8}$$

Where $F(\alpha x)=1$ when $\alpha=0$ and decreases with x, $\alpha\geq 0$ is the reabsorption coefficient and is a constant, and Q_o is the flux across the cross-section at x=0.

The boundary conditions (6) and (7) are well known Beavers and Joseph[1] condition when applied to tangential velocity.

We introduce the stream function ψ such that

$$u=\frac{\partial\psi}{\partial y} \quad , \quad v=-\frac{\partial\psi}{\partial x} \tag{9}$$

And the non-dimensional quantities as

$$x'=\frac{x}{\lambda},\; y'=\frac{y}{d},\; \eta_1'=\frac{\eta_1}{d},\quad \eta_2'=\frac{\eta_2}{d},\; \psi'=\frac{\psi}{Q_o},\; \alpha'=\alpha\lambda,\; p'=\frac{d^2}{\mu Q_o}p$$

Where $d = d_1 + d_2$.

By introducing the above non-dimensional variables the equations (3)—(5) can be written as (the primes are dropped)

$$\left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \psi = \delta \mathrm{R}\left[\frac{\partial \psi}{\partial y}\left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x}\left(\delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\frac{\partial \psi}{\partial y}\right] \quad (10)$$

Where $\delta = \frac{d}{\lambda}$ and $R = \frac{Q_o}{\nu}$.

Further, the boundary conditions (6—8) becomes

$$\left[\frac{\partial \psi}{\partial y} + A\delta \sin(2\pi x)\frac{\partial \psi}{\partial x}\right] = -\xi\left[\frac{\partial^2 \psi}{\partial y^2} - A\delta \sin(2\pi x)\frac{\partial^2 \psi}{\partial y \partial x}\right] \text{ at } y = \eta_1(x) = \beta_1 + \varepsilon_1 \cos(2\pi x) \quad (11)$$

$$\left[\frac{\partial \psi}{\partial y} - B\delta \sin(2\pi x + \phi)\frac{\partial \psi}{\partial x}\right] = -\xi\left[\frac{\partial^2 \psi}{\partial y^2} - B\delta \sin(2\pi x + \phi)\frac{\partial^2 \psi}{\partial y \partial x}\right] \text{ at } y = \eta_2(x) = \beta_2 + \varepsilon_2 \cos(2\pi x + \phi) \quad (12)$$

$$\psi = \frac{1}{2}F(\alpha x) \text{ at } \quad y = \eta_1(x) = \beta_1 + \varepsilon_1 \cos(2\pi x) \quad (13)$$

$$\psi = -\frac{1}{2}F(\alpha x) \text{ at } y = \eta_2(x) = \beta_2 + \varepsilon_2 \cos(2\pi x + \phi) \quad (14)$$

Where $A = \left(-\frac{2\pi a_1}{\lambda}\right)$, $B = \left(\frac{2\pi b_1}{\lambda}\right), \varepsilon_1 = \frac{a_1}{d}, \varepsilon_2 = -\frac{b_1}{d}, \beta_1 = \frac{d_1}{d}, \beta_2 = -\frac{d_2}{d}, \xi = \frac{\sqrt{\gamma}}{d\beta}$

The parameter R is the Reynolds number and δ is the wavenumber (the ratio of inlet width to the wavelength). ε_1 and ε_2 are amplitude ratios (the ratios of amplitudes a_1 and b_1 to the inlet width respectively) and β_1 and β_2 are ratios of distance from the x-axis to the upper wall and lower wall to the inlet width respectively. In this problem, we consider exponentially decaying bulk flow [6] that is, in equation (8), *F* is taken as

$$F(\alpha x) = e^{-\alpha x} \quad (15)$$

3 METHOD OF SOLUTION

It is observed that the flow is quite complex because of nonlinearity of governing equation and the boundary conditions (10)-(14). Thus to solve equation (10) for velocity components, in the present analysis, we assume the wave number δ (long wavelength approximation). We shall seek a solution for stream function $\psi(x, y)$ in the form of a power series in terms of δ , as

$$\psi(x, y) = \psi_0(x, y) + \delta \psi_1(x, y) + .. \quad (16)$$

Substituting equation (16) in equations (10)-(14), and equating the coefficients of like powers of δ , we get the following sets of equations for $\psi_0(x, y)$, $\psi_1(x, y)$,...

Zeroth Order System

$$\frac{\partial^4 \psi_0}{\partial y^4} = 0 \tag{17}$$

The corresponding boundary conditions are:

$$\frac{\partial \psi_0}{\partial y} = -\xi \frac{\partial^2 \psi_0}{\partial y^2} \text{ at } y = \eta_1(x) \text{ and } y = \eta_2(x) \tag{18}$$

$$\psi_o = F(\alpha x) = \frac{1}{2} e^{-\alpha x} \qquad at\ \ y = \eta_1(x) \tag{19}$$

$$\psi_o = -F(\alpha x) = -\frac{1}{2} e^{-\alpha x} \quad at\ \ y = \eta_2(x) \tag{20}$$

First Order System

$$\frac{\partial^4 \psi_1}{\partial y^4} = \mathrm{R}\left[\frac{\partial \psi_0}{\partial y}\frac{\partial^3 \psi_0}{\partial y^2 \partial x} - \frac{\partial \psi_0}{\partial x}\frac{\partial^3 \psi_0}{\partial y^3}\right] \tag{21}$$

The corresponding boundary conditions are:

$$\frac{\partial \psi_1}{\partial y} = A\sin(2\pi x)\frac{\partial \psi_0}{\partial x} - \xi\left[\frac{\partial^2 \psi_1}{\partial y^2} - A\sin(2\pi x)\frac{\partial^2 \psi_0}{\partial y \partial x}\right] \qquad \text{at } y = \eta_1(x) \tag{22}$$

$$\frac{\partial \psi_1}{\partial y} = B\sin(2\pi x + \phi)\frac{\partial \psi_0}{\partial x} - \xi\left[\frac{\partial^2 \psi_1}{\partial y^2} - B\sin(2\pi x + \phi)\frac{\partial^2 \psi_0}{\partial y \partial x}\right] \qquad \text{at } y = \eta_2(x) \tag{23}$$

$$\psi_0 = 0 \text{ at } y = \eta_1(x) \qquad \text{and} \quad y = \eta_2(x) \tag{24}$$

Similar expressions can be written for higher orders of δ. However, since we are looking for an approximate analytical solution for the problem, we consider up to the order of δ^1 equations.

The solution of equation (17) along with the corresponding boundary conditions (18-20) as

$$\psi_0 = \frac{1}{2}\left[A_1(x)y^3 + A_2(x)y^2 + A_3(x)y + A_4(x)\right] \tag{25}$$

Following the similar procedure as in equation (25) the solution of equation (21) along with boundary conditions (22-24) is

$$\psi_1 = R\left[\frac{1}{840}A_5(x)y^7 + \frac{1}{360}A_6(x)y^6 + \frac{1}{120}A_7(x)y^5 + \frac{1}{24}A_8(x)y^4\right]$$
$$+\frac{1}{6}A_9(x)y^3 + \frac{1}{2}A_{10}(x)y^2 + A_{11}(x)y + A_{12}(x) \tag{26}$$

By substituting the value of ψ_0 and ψ_1 in equation (16), we get

$$\psi = \frac{1}{2}\left[A_1(x)y^3 + A_2(x)y^2 + A_3(x)y + A_4(x)\right]$$
$$+\delta\left[R\left(\frac{1}{840}A_5(x)y^7 + \frac{1}{360}A_6(x)y^6 + \frac{1}{120}A_7(x)y^5 + \frac{1}{24}A_8(x)y^4\right)\right.$$

$$+\frac{1}{6}A_9(x)y^3+\frac{1}{2}A_{10}(x)y^2+A_{11}(x)y+A_{12}(x)\Bigg] \tag{27}$$

Now, the nondimensional pressure $p(x,y)$ can be obtained by using equations (27), (9) and (4), and it is given as

$$p(x,y)=\delta\frac{\partial u}{\partial x}+\frac{1}{\delta}\int\frac{\partial^2 u}{\partial y^2}dx-R\left(\int u\frac{\partial u}{\partial x}dx+\int v\frac{\partial u}{\partial y}dx\right) \tag{28}$$

The mean pressure is given as

$$\overline{p}(x)=\frac{1}{\eta_1(x)-\eta_2(x)}\int_{\eta_2(x)}^{\eta_1(x)}p(x,y)\,dy \tag{29}$$

Further, the mean pressure drop between $x=0$ and $x=x_0$ is

$$\Delta\overline{p}(x_0)=\overline{p}(0)-\overline{p}(x_0) \tag{30}$$

The wall shear stress $\tau_w(x)$ is defined as

$$\tau_w(x)=\frac{\left(\sigma_{yy}-\sigma_{xx}\right)\frac{dy}{dx}+\sigma_{xy}\left[1-\left(\frac{dy}{dx}\right)^2\right]}{1+\left(\frac{dy}{dx}\right)^2} \qquad \text{at } y=\eta_1(x) \text{ and } y=\eta_1(x) \tag{31}$$

Where $\sigma_{xx}=2\mu\frac{\partial u}{\partial x},\ \sigma_{yy}=2\mu\frac{\partial v}{\partial y},$ and $\sigma_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)$

Using the non-dimensional quantity $\tau'_{w_1}=\frac{d^2}{\mu Q_0}\tau_{w_1}$ and $\tau'_{w_2}=\frac{d^2}{\mu Q_0}\tau_{w_2}$, the wall shear stress τ_{w_1} and τ_{w_2} (after dropping the prime) can be written as

$$\tau_{w_1}=\frac{2\delta^2\left(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)\frac{d\eta_1}{dx}+\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)\left[1-\delta^2\left(\frac{d\eta_1}{dx}\right)^2\right]}{1+\delta^2\left(\frac{d\eta_1}{dx}\right)^2} \tag{32}$$

$$\tau_{w_2}=\frac{2\delta^2\left(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)\frac{d\eta_2}{dx}+\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)\left[1-\delta^2\left(\frac{d\eta_2}{dx}\right)^2\right]}{1+\delta^2\left(\frac{d\eta_2}{dx}\right)^2} \tag{33}$$

It may be noted that in equation (28), the integrals are difficult to evaluate analytically to get closed form expression for $p(x,y)$. Therefore, they are calculated by numerical integration.

4 RESULTS AND DISCUSSION

The purpose of the present discussion is to analyze the behavior of a steady incompressible fluid flow in an asymmetric channel of slowly varying cross-section with absorbing wall by considering a slip velocity at the walls.

It may be noted that ϕ characterized the phase difference which varies in the range $0 \le \phi \le \pi$. Here $\phi = 0$ represents symmetric channel. $\phi = \pi$ represents the asymmetric channel with waves are in phase. α and ξ represents reabsorption coefficient and slip at the channel wall, respectively. It is observed that in the absence of slip i.e., $\xi \to 0$, our results are in tune with those of Muthu and Tesfahun[10].

We discuss the effect of these parameters on the transverse velocity $v(x, y)$, mean pressure drop $(\Delta \overline{p})$, and wall shear stress (τ_w). The following parameters are fixed as A=-0.0628, B=0.0628, $\beta_1 = 0.5,\ \beta_2 = -0.5,\ \varepsilon_1 = 0.1,\ \varepsilon_2 = -0.1,\ \delta = 0.1$ in our numerical calculation. For low Reynolds number flow, we have taken $R = 1.0$. To see the effect of ξ we have taken $\xi = 0,\ 0.15$ and 0.4.

The Transverse velocity v:

The transverse velocity $v(x, y)$ which is obtained from equations (9) and (27) Here we have discussed the effects of the phase difference (ϕ), in the presence of non-zero slip coefficient (ξ) on the transverse velocity by taking the behavior at a different cross-section of the channel. We have taken $x = 0.1, 0.5, 0.9$ and $\phi = 0, \frac{\pi}{2}, \pi$.

Figure 2(a) displays the effect of (ϕ) on *v* at *x*=0.1 and ξ=0.0. It may be observed that as (ϕ) increases from 0 to π, the magnitude of *v* decreases. It may be remarked that the reabsorption value at the wall is fixed at *x*=0.1 and when (ϕ) increases, the cross-sectional area is reduced. This results in lesser *v* values. Now, if ξ=0.15 similar effect is observed as above. When (ϕ) varies from 0 to π mixed trends is observed in velocity.

If ξ=0.4 the velocity decreases when ϕ varies from 0 to π. But comparing with no slip (ξ=0.0) case velocity increases in quantity. This may be due to the effect of the slip (see figures 2(b), 2(c)).

Figure 3(a) displays the effect of ϕ on *v* at *x*=0.5 with ξ=0.0. It may be noted that as ϕ increases the magnitude of *v* has mixed trends, due to the variation of the cross-section of the channel at *x*=0.5.

If ξ=0.15 and ξ=0.4, similar mixed trends is observed on *v*, due to the effect of slip (see figures 3(b), 3(c)).Figures 4(a)-4(c) display the effect of ϕ on *v* when *x*=0.9 for ξ=0.0, 0.15 and 0.4. It is observed that as ϕ increases the magnitude of *v* has a mixed trend.

Mean Pressure drop

The value of the mean pressure drop (29) over the length of the channel is calculated from different values of ϕ and ξ. Figure 5(a) represents the effect of ϕ when ξ=0.0. It is observed that as the width of channel contracts, the mean pressure drop increases. Particularly, at the entrance of the channel, the mean pressure drop for the asymmetrical channel is more than the symmetrical channel.

It can be understood from figure 5(a) for $\phi = 0, \frac{\pi}{2}, \pi$. However, due to contraction in the middle of the channel, the reverse is true at the end of the channel.

When ξ =0.15, a similar trend as mentioned above is observed, with a quality difference (see fig.5 (b)).

As ξ =0.4, the trend is reversed, this shows the effect of slip combined with asymmetry nature of the channel (fig. 5(c)).

Magnitude of wall shear stress

The effects of ϕ and ξ on the magnitude of the wall shear stress ($|\tau_{w_1}|$ and $|\tau_{w_2}|$) are presented in figures 6 and 7 respectively.

It may be noted from figures 6(a) to 6(b), and 7(a) to 7(b) that the upper wall and lower on shear stress (in magnitude) increases as the channel changed from symmetry to asymmetry and no-slip to slip conditions, except in the middle of the channel where there is more contraction.

This indicates that as the width of the channel decreases due to asymmetry nature of walls, ($|\tau_{w_1}|$ and $|\tau_{w_2}|$) increases. But when ξ =0.4, the nature of the curve is oscillatory [See fig. 6(c) and 7 (c)].

5 CONCLUSIONS

The main contribution of this study is to see the effect of the phase difference in the presence of slip at the walls on the flow of incompressible fluid in an asymmetric channel of the slowly varying cross-section. The mathematical problem is solved using a regular perturbation method assuming the ratio of inlet width to wavelength is small. We observe the following observation in the present study.

(i) As phase difference increases the magnitude of velocity decreases.
(ii) As the channel changes from symmetric to asymmetric the mean pressure drop increases.
(iii) The wall shear stress increases as the channel changes from symmetry to asymmetry and no slip to slip.

APPENDIX – IMPORTANT FORMULAS AND GRAPHS

$$A_1(x) = \frac{4e^{-\alpha x}}{\eta_2^3-\eta_1^3+3\eta_1^2\eta_2-3\eta_2^2\eta_1-12\xi^2\eta_2+12\xi^2\eta_1}, \quad A_2(x)=\frac{-6\left(\eta_1+2\xi+\eta_2\right)e^{-\alpha x}}{\eta_2^3-\eta_1^3+3\eta_1^2\eta_2-3\eta_2^2\eta_1-12\xi^2\eta_2+12\xi^2\eta_1}$$

$$A_3(x)=\frac{12\left(\xi\eta_1+2\xi^2+\eta_1\eta_2+\xi\eta_2\right)e^{-\alpha x}}{\eta_2^3-\eta_1^3+3\eta_1^2\eta_2-3\eta_2^2\eta_1-12\xi^2\eta_2+12\xi^2\eta_1}$$

$$A_4(x)=\frac{-e^{-\alpha x}\left(-\eta_2^3-\eta_1^3+3\eta_1^2\eta_2+3\eta_2^2\eta_1+12\xi\eta_2+12\xi\eta_1+12\xi\eta_1\eta_2\right)}{\eta_2^3-\eta_1^3+3\eta_1^2\eta_2-3\eta_2^2\eta_1-12\xi^2\eta_2+12\xi^2\eta_1}$$

$$A_5(x)=12A_1(x)\left(\frac{dA_1}{dx}\right),\ A_6(x)=12A_2(x)\left(\frac{dA_1}{dx}\right)$$

$$A_7(x)=4A_2(x)\left(\frac{dA_2}{dx}\right)+6A_3(x)\left(\frac{dA_1}{dx}\right)-6A_1(x)\left(\frac{dA_3}{dx}\right),$$

$$A_8(x)=2A_3(x)\left(\frac{dA_2}{dx}\right)-6A_1(x)\left(\frac{dA_4}{dx}\right)$$

$$A_9(x)=\frac{1}{420(\eta_1-\eta_2)\left(12\xi^2-(\eta_1-\eta_2)^2\right)}\Big[-7560B(2\xi+\eta_1-\eta_2)A_{13}\sin(2\pi x+\phi)$$

$$+7560(2\xi-\eta_1+\eta_2)A_{14}\sin(2\pi x)$$

$$-3024R(\eta_1\text{-}\eta_2)\Big\{\xi\Big(A_1A_{15}+\left(10(\eta_1+\eta_2)(\eta_1^2+\eta_2^2)\xi-5\eta_2^4-5\eta_1^4\right)\frac{A_2}{6}$$

$$+\frac{5}{6}\left(2\xi\left(\eta_2^2+\eta_1^2+\eta_1\eta_2\right)-\eta_1^3-\eta_2^3\right)A_3\Big)\frac{dA_1}{dx}$$

$$+\frac{10\xi A_{17}}{9}\frac{dA_2}{dx}-\frac{5\xi A_1}{6}\left(2\xi\left(\eta_2^2+\eta_1^2+\eta_1\eta_2\right)-\eta_1^3-\eta_2^3\right)\frac{dA_3}{dx}$$

$$-\frac{5\xi A_1}{2}\left(2\xi(\eta_1+\eta_2)-\eta_1^2-\eta_2^2\right)\frac{dA_4}{dx}+A_5A_{16}$$

$$+\left(\left(\eta_1^4+\eta_2^4+\eta_1^3\eta_2+\eta_2^3\eta_1+\eta_1^2\eta_2^2\right)\frac{\xi}{36}-\frac{1}{216}(\eta_1\text{-}\eta_2)^2(\eta_1+\eta_2)\left(2\eta_2^2+2\eta_1^2+\eta_1\eta_2\right)\right)A_5$$

$$+\left(\frac{5}{72}(\eta_1+\eta_2)\left(\eta_1^2+\eta_2^2\right)\xi-\frac{1}{144}(\eta_1\text{-}\eta_2)^2\left(3\eta_2^2+3\eta_1^2+4\eta_1\eta_2\right)\right)A_7$$

$$+\frac{5}{72}\left(4\xi\left(\eta_2^2+\eta_1^2+\eta_1\eta_2\right)-(\eta_1+\eta_2)(\eta_1\text{-}\eta_2)^2\right)A_8\Big\}\Big]$$

$$A_{13}(x)=\eta_2^2\left(\frac{1}{3}\eta_2+\xi\right)\frac{dA_1}{dx}+\frac{2}{3}\eta_2\left(\frac{1}{2}\eta_2+\xi\right)\frac{dA_2}{dx}+\frac{1}{3}(\eta_2+\xi)\frac{dA_3}{dx}+\frac{1}{3}\frac{dA_4}{dx}$$

$$A_{14}(x)=\eta_1^2\left(\frac{1}{3}\eta_1+\xi\right)\frac{dA_1}{dx}+\frac{2}{3}\eta_1\left(\frac{1}{2}\eta_1+\xi\right)\frac{dA_2}{dx}+\frac{1}{3}(\eta_1+\xi)\frac{dA_3}{dx}+\frac{1}{3}\frac{dA_4}{dx}$$

$$A_{15}(x)=\left(\eta_1^4+\eta_2^4+\eta_1^2\eta_2^2+\eta_1^3\eta_2+\eta_1\eta_2^3\right)\xi-\frac{1}{2}\left(\eta_1^5+\eta_2^5\right)$$

$$A_{16}(x)=\frac{1}{72}(\eta_1+\eta_2)\left(\eta_2^2+\eta_1^2+\eta_1\eta_2\right)\left(\eta_1^2+\eta_2^2-\eta_1\eta_2\right)\xi$$

$$-\frac{5}{1008}\left(\eta_1^4+\frac{8}{5}\eta_1^3\eta_2+\frac{9}{5}\eta_2^2\eta_1^2+\frac{8}{5}\eta_2^3\eta_1+\eta_2^4\right)\left(\eta_1^2-\eta_2^2\right)^2$$

$$A_{17}(x)=\left(\left(\eta_1^2+\eta_2^2+\eta_1\eta_2\right)\xi-\frac{1}{2}\eta_1^3\text{-}\frac{1}{2}\eta_2^2\right)A_2(x)-3A_3(x)$$

$$A_{10}(x)=\frac{1}{420}\frac{1}{(\eta_1-\eta_2)\left(-\eta_1^2+2\eta_2\eta_1-\eta_2^2+12\xi^2\right)}\left[15120B\left\{\frac{1}{3}\eta_1^2+\left(-\frac{1}{6}\eta_2+\xi\right)\eta_1-\frac{1}{6}\eta_1^2\right\}A_{18}\sin(2\pi x+\phi)\right.$$

$$\left.-15120\left(-\frac{1}{6}\eta_1^2-\frac{1}{6}\eta_1\eta_2+\eta_2\left(\frac{1}{3}\eta_2+\xi\right)\right)AA_{19}\sin(2\pi x)\right.$$

$$-15120R(\eta_1-\eta_2)\left\{-\frac{1}{5}\xi\left(A_{21}A_1+A_{27}A_2+\frac{5}{3}A_3A_{28}\right)\frac{dA_1}{dx}+\frac{2}{3}\xi\left(A_3\left(\xi+\frac{\eta_1}{2}+\frac{\eta_2}{2}\right)-\frac{A_{28}A_2}{3}\right)\frac{dA_2}{dx}\right.$$

$$+A_1\xi\left(-\frac{1}{6}\eta_1^3-\frac{1}{3}\eta_2\eta_1^2+\eta_2\left(-\frac{1}{3}\eta_2+\xi\right)\eta_1-\frac{1}{6}\eta_2^3\right)\frac{dA_4}{dx}$$

$$+\frac{1}{3}A_1A_{28}\xi\frac{dA_3}{dx}+\frac{1}{3780}A_5\eta_1^7+\left(-\frac{1}{2520}A_5\xi+\frac{1}{2160}A_6+\frac{1}{1890}\eta_2A_5\right)\eta_1^6$$

$$\left.-\frac{1}{5}A_{20}\eta_1^5-\frac{1}{5}A_{22}\eta_1^4-\frac{1}{5}A_{23}\eta_1^3-\frac{1}{315}A_{24}\eta_2\eta_1^2-\frac{1}{315}A_{25}\eta_1\eta_2^2-\frac{1}{2520}A_{26}\eta_2^3\right\}\Bigg]$$

$$A_{18}(x)=\frac{1}{3}\left[\left(\eta_2^3+3\eta_2^2\xi\right)\frac{dA_1}{dx}+\left(\eta_2^2+2\eta_2\xi\right)\frac{dA_2}{dx}+\left(\eta_2+\xi\right)\frac{dA_3}{dx}+\frac{dA_4}{dx}\right]$$

$$A_{19}(x)=\frac{1}{3}\left[\left(\eta_1^3+3\eta_2^2\xi\right)\frac{dA_1}{dx}+\left(\eta_1^2+2\eta_1\xi\right)\frac{dA_2}{dx}+\left(\eta_1+\xi\right)\frac{dA_3}{dx}+\frac{dA_4}{dx}\right]$$

$$A_{20}(x)=\frac{1}{1512}\left[\left(24\xi A_5-7A_6\right)\eta_2+7\xi A_6-7A_7+3A_5\eta_2^2\right]$$

$$A_{21}(x)=-\frac{\eta_2^6}{6}-+\frac{\eta_2}{6}\left(-2\eta_1^5+6\xi\eta_1^4\right)-\frac{\eta_1^6}{6}-\frac{\eta_2^5\eta_1}{3}+\eta_2^4\eta_1\xi+\eta_1^2\eta_2^3\xi+\eta_1^3\eta_2^2\xi$$

$$A_{22}(x)=\frac{A_5\eta_2^3}{504}+\left(\frac{\xi A_5}{63}+\frac{A_6}{216}\right)\eta_2^2+\left(-\frac{A_7}{108}+\frac{7\xi A_6}{216}\right)\eta_2+\frac{\xi A_7}{72}-\frac{5A_8}{432}$$

$$A_{23}(x)=\frac{A_5\eta_2^4}{504}+\frac{1}{1512}\left(24\xi A_5+7A_6\right)\eta_2^3+\frac{1}{1512}\left(49\xi A_6+21A_7\right)\eta_2^2+\frac{1}{1512}\left(126\xi A_7-35A_8\right)\eta_2+\frac{5}{72}\xi A_8$$

$$A_{24}(x)=\frac{A_5\eta_2^4}{8}+\frac{1}{24}\left(24\xi A_5+7A_6\right)\eta_2^3+\frac{1}{24}\left(49\xi A_6+21A_7\right)\eta_2^2+\frac{1}{24}\left(126\xi A_7+105A_8\right)\eta_2+\frac{175}{8}\xi A_8$$

$$A_{25}(x)=-\frac{A_5\eta_2^4}{6}+\frac{1}{24}\left(24\xi A_5-7A_6\right)\eta_2^3+\frac{1}{24}\left(49\xi A_6-14A_7\right)\eta_2^2+\frac{1}{24}\left(126\xi A_7-35A_8\right)\eta_2+\frac{175}{8}\xi A_8$$

$$A_{26}(x)=-\frac{2A_5\eta_2^4}{3}+\frac{1}{6}\left(6\xi A_5-7A_6\right)\eta_2^3+\frac{1}{6}\left(14\xi A_6-14A_7\right)\eta_2^2+\frac{1}{6}\left(42\xi A_7-35A_8\right)\eta_2+35\xi A_8$$

$$A_{27}(x)=-\frac{5\eta_2^4}{18}-\frac{5}{9}\eta_2^4\eta_1+\frac{5}{3}\eta_2^3\eta_1\xi+\frac{5}{3}\eta_1^2\eta_2^2\xi+\frac{1}{18}\left(-10\eta_1^4+30\eta_1^3\xi\right)\eta_2-\frac{5}{18}\eta_1^5$$

$$A_{28}(x)=-\frac{1}{6}\eta_2^4-\frac{1}{3}\eta_2^3\eta_1+\eta_2^2\eta_1\xi+\frac{1}{6}\left(-2\eta_1^3+6\eta_1^2\xi\right)\eta_2-\frac{1}{6}\eta_1^4$$

$$A_{11}(x)=\frac{1}{840}\frac{12}{(\eta_1-\eta_2)\left(-\eta_1^2+2\eta_1\eta_2-\eta_2^2+\xi^2\right)}$$

$$\Big[-10080BB_1B_2\sin(2\pi x+\phi)-5040AB_{10}B_3\sin(2\pi x)$$

$$+1008R(\eta_1-\eta_2)\left\{B_{12}\xi\frac{dA_1}{dx}+\frac{10}{7}B_7\xi\frac{dA_2}{dx}-\frac{5}{3}A_1\xi B_8\frac{dA_3}{dx}\right.$$

$$-5A_1\xi B_9\frac{dA_4}{dx}+A_5\left(\left(\frac{1}{126}\xi+\frac{1}{126}\eta_2\right)\eta_1^7+\left(-\frac{1}{84}\xi^2+\frac{1}{1008}\eta_2^2-\frac{13}{504}\xi\eta_2\right)\eta_1^6\right.$$

$$-\frac{1}{84}\eta_2\left(\xi^2+\frac{1}{2}\eta_2^2+\frac{9}{2}\xi\eta_2\right)\eta_1^5-\frac{1}{84}\eta_2^2\left(\xi^2+\frac{1}{2}\eta_2^2+\frac{9}{2}\xi\eta_2\right)\eta_1^4$$

$$-\frac{1}{84}\eta_2^3\left(\xi^2+\frac{1}{2}\eta_2^2+\frac{9}{2}\xi\eta_2\right)\eta_1^3-\frac{1}{84}\eta_2^4\left(\xi^2-\frac{1}{12}\eta_2^2+\frac{9}{2}\xi\eta_2\right)\eta_1^2$$

$$-\frac{1}{84}\eta_2^5\left(\xi^2-\frac{2}{3}\eta_2^2+\frac{13}{6}\xi\eta_2\right)\eta_1-\frac{1}{84}\eta_2^6\left(\xi-\frac{2}{3}\eta_2\right)\xi\Bigg)$$

$$+A_6\left(\left(\frac{1}{72}\xi+\frac{1}{72}\eta_2\right)\eta_1^6-\frac{1}{36}\xi\left(2\eta_2+\xi\right)\eta_1^5-\frac{1}{36}\left(\frac{1}{2}\eta_2^2+4\xi\eta_2+\xi^2\right)\eta_2\eta_1^4\right.$$

$$-\frac{1}{36}\left(\frac{1}{2}\eta_2^2+4\xi\eta_2+\xi^2\right)\eta_2^2\eta_1^3-\frac{1}{36}\xi\eta_2^3\left(4\eta_2+\xi\right)\eta_1^2$$

$$\left.-\frac{1}{36}\left(2\eta_2\xi-\frac{1}{2}\eta_2^2+\xi^2\right)\eta_2^4\eta_1-\frac{1}{36}\left(-\frac{1}{2}\eta_2+\xi\right)\xi\eta_2^5\right)+B_{11}A_7-\frac{5}{12}B_4A_8\Big\}\Big]$$

$$B_1(x)=\eta_2^2\left(\frac{1}{3}\eta_2+\xi\right)\frac{dA_1}{dx}+\frac{2}{3}\eta_2\left(\frac{1}{2}\eta_2+\xi\right)\frac{dA_2}{dx}+\left(\frac{1}{3}\eta_2+\frac{1}{3}\xi\right)\frac{dA_3}{dx}+\frac{1}{3}\frac{dA_4}{dx}$$

$$B_2(x)=\frac{1}{4}\eta_1^3+\left(\frac{1}{4}\eta_2+\xi\right)\eta_1^2+\left(-\frac{1}{2}\eta_2+\xi\right)\eta_1\eta_2-\frac{1}{2}\eta_2^2\xi$$

$$B_3(x)=\eta_1^2\left(\frac{1}{3}\eta_2+\xi\right)\frac{dA_1}{dx}+\frac{2}{3}\eta_1\left(\frac{1}{2}\eta_1+\xi\right)\frac{dA_2}{dx}+\left(\frac{1}{3}\eta_1+\frac{1}{3}\xi\right)\frac{dA_3}{dx}+\frac{1}{3}\frac{dA_4}{dx}$$

$$B_4(x)=\eta_1^4\left(-\frac{1}{6}\eta_2-\frac{1}{6}\xi\right)+\eta_1^3\left(\xi^2+\frac{5}{3}\xi\eta_2+\frac{1}{6}\eta_2^2\right)+\eta_2\eta_1^2\left(3\xi\eta_2+\frac{1}{6}\eta_2^2+\xi^2\right)$$

$$+\eta_1\eta_2^2\left(\frac{5}{3}\xi\eta_2-\frac{1}{6}\eta_2^2+\xi^2\right)+\xi\eta_2^3\left(-\frac{1}{6}\eta_2+\xi\right)$$

$$B_5(x)=\left(\eta_2+\xi\right)\eta_1^6-\left(-\frac{1}{2}\eta_2+\xi\right)\eta_2\eta_1^5-3\eta_2^2\eta_1^4\xi-3\eta_1^3\eta_2^3\xi-3\left(-\frac{1}{6}\eta_2+\xi\right)\eta_2^4\eta_1^2-\eta_2^5\left(-\eta_2+\xi\right)\eta_1+\eta_2^6\xi$$

$$B_6(x)=\left(\frac{5}{3}\xi+\frac{5}{3}\eta_2\right)\eta_1^5-\frac{5}{3}\left(-\frac{1}{2}\eta_2+\xi\right)\eta_2\eta_1^4-5\eta_2^2\eta_1^3\xi-5\left(-\frac{1}{6}\eta_2+\xi\right)\eta_1^2\eta_2^3-\frac{5}{3}\eta_2^4\left(-\eta_2+\xi\right)\eta_1+\frac{5}{3}\eta_2^5\xi$$

$$B_7(x)=\left(\left(\eta_2+\xi\right)\eta_1^4-\left(-\frac{1}{2}\eta_2+\xi\right)\eta_2\eta_1^3-3\left(-\frac{1}{6}\eta_2+\xi\right)\eta_2^2\eta_1^2-\eta_2^3\left(-\eta_2+\xi\right)\eta_1+\xi\eta_2^4\right)A_2$$

$$+9A_3\left(\frac{1}{6}\eta_1^2+\left(\xi+\frac{2}{3}\eta_2\right)\eta_1+\left(\frac{1}{6}\eta_2+\xi\right)\eta_2\right)$$

$$B_8(x)=\left(\eta_2+\xi\right)\eta_1^4-\left(-\frac{1}{2}\eta_2+\xi\right)\eta_2\eta_1^3-3\left(-\frac{1}{6}\eta_2+\xi\right)\eta_2^2\eta_1^2-\eta_2^3\left(-\eta_2+\xi\right)\eta_1+\xi\eta_2^4$$

$$B_9(x)=\left(\eta_2+\xi\right)\eta_1^3-\left(-\eta_2+\xi\right)\eta_2\eta_1^2-\left(-\eta_2+\xi\right)\eta_2^2\eta_1-\eta_2^3\xi$$

$$B_{10}(x)=\left(\eta_2+\xi\right)\eta_1^2-2\left(\frac{1}{4}\eta_2+\xi\right)\eta_2\eta_1-2\left(\frac{1}{4}\eta_2+\xi\right)\eta_2^2$$

$$B_{11}(x)=\frac{1}{36}\left(\eta_2+\xi\right)\eta_1^5-\left(-\frac{11}{72}\xi\eta_2-\frac{1}{144}\eta_2^2-\frac{1}{12}\xi^2\right)\eta_1^4-\frac{1}{12}\left(\frac{7}{2}\xi\eta_2+\xi^2+\frac{1}{2}\eta_2^2\right)\eta_1^3\eta_2$$

$$-\frac{1}{12}\eta_2^2\eta_1^2\left(\frac{1}{12}\eta_2^2+\frac{7}{2}\xi\eta_2+\xi^2\right)-\frac{1}{12}\left(2\eta_2+\xi\right)\left(-\frac{1}{6}\eta_2+\xi\right)\eta_2^3\eta_1-\frac{1}{12}\left(-\frac{1}{3}\eta_2+\xi\right)\xi\eta_2^4$$

$$B_{12}(x)=B_5A_1+B_6A_2+\frac{5}{3}\left[\left(\eta_2+\xi\right)\eta_1^4-\left(-\frac{1}{2}\eta_2+\xi\right)\eta_2\eta_1^3-3\left(-\frac{1}{6}\eta_2+\xi\right)\eta_2^2\eta_1^2-\eta_2^3\left(-\eta_2+\xi\right)\eta_1+\xi\eta_2^4\right]A_3$$

$$C_1(x)=\eta_2^2\left(\frac{1}{3}\eta_2+\xi\right)\frac{dA_5}{dx}+\frac{2}{3}\eta_2\left(\frac{1}{2}\eta_2+\xi\right)\frac{dA_6}{dx}+\left(\frac{1}{3}\eta_2+\frac{1}{3}\xi\right)\frac{dA_7}{dx}+\frac{1}{3}\frac{dA_8}{dx}$$

$$C_2(x)=\eta_1^2\left(\frac{1}{3}\eta_1+\xi\right)\frac{dA_5}{dx}+\frac{2}{3}\eta_1\left(\frac{1}{2}\eta_1+\xi\right)\frac{dA_6}{dx}+\left(\frac{1}{3}\eta_1+\frac{1}{3}\xi\right)\frac{dA_7}{dx}+\frac{1}{3}\frac{dA_8}{dx}$$

$$C_3(x)=-\frac{2}{5}A_5\eta_2^4+\left(\xi A_5-\frac{7}{6}A_6\right)\eta_2^3+\left(\frac{7}{3}\xi A_6-\frac{7}{3}A_7\right)\eta_2^2+\left(7\xi A_7-\frac{35}{6}A_8\right)\eta_2+35\xi A_8$$

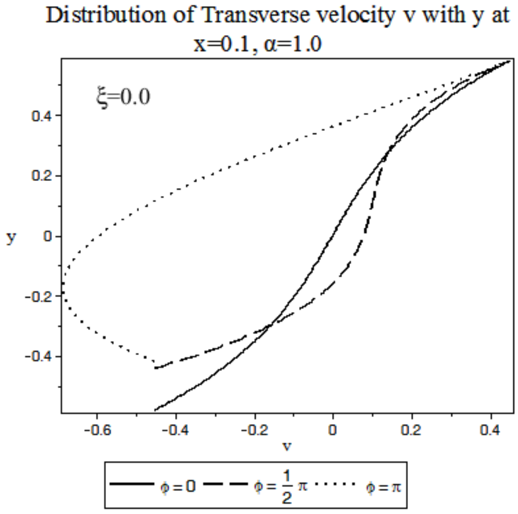
$$C_4(x)=\left(\left(\xi+\frac{1}{2}\eta_2\right)\eta_1^3-\xi\eta_2\eta_1^2-\left(-\frac{1}{2}\eta_2+\xi\right)\eta_1\eta_2^2+\xi\eta_2^3\right)A_6+9\left(\frac{1}{6}\eta_2+\xi+\frac{1}{6}\eta_1\right)A_7$$

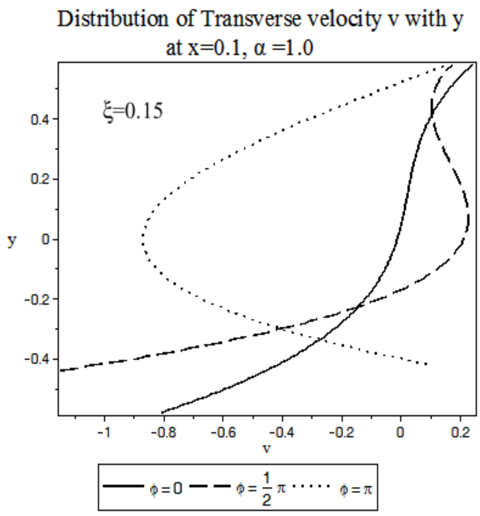
$$C_5(x)=\left(\xi+\frac{1}{2}\eta_2\right)\eta_1^5-\xi\eta_2\eta_1^4-\xi\eta_2^2\eta_1^3-\xi\eta_1^2\eta_2^3-\eta_2^4\left(\xi+\frac{1}{2}\eta_2\right)\eta_1+\xi\eta_2^5$$

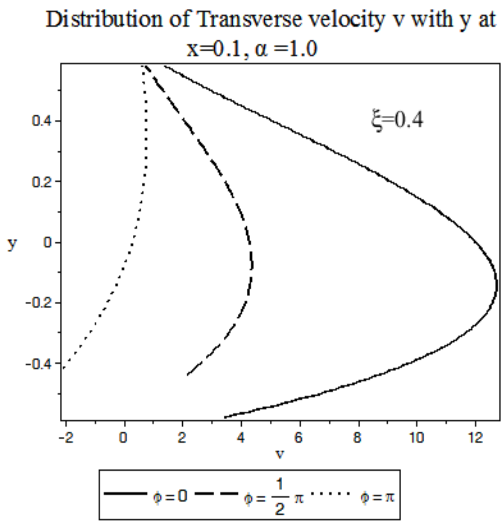
$$C_6(x)=\left(\xi+\frac{1}{2}\eta_2\right)\eta_1^3-\xi\eta_2\eta_1^2-\eta_2^2\left(\xi-\frac{1}{2}\eta_2\right)\eta_1+\xi\eta_2^3$$

$$C_7(x)=-\frac{1}{7}A_5\eta_2^4+\left(\xi A_5-\frac{1}{4}A_6\right)\eta_2^3+\eta_2^2\left(2\xi A_6-\frac{1}{2}A_7\right)+\left(5\xi A_7-\frac{5}{4}A_8\right)\eta_2+20\xi A_8$$

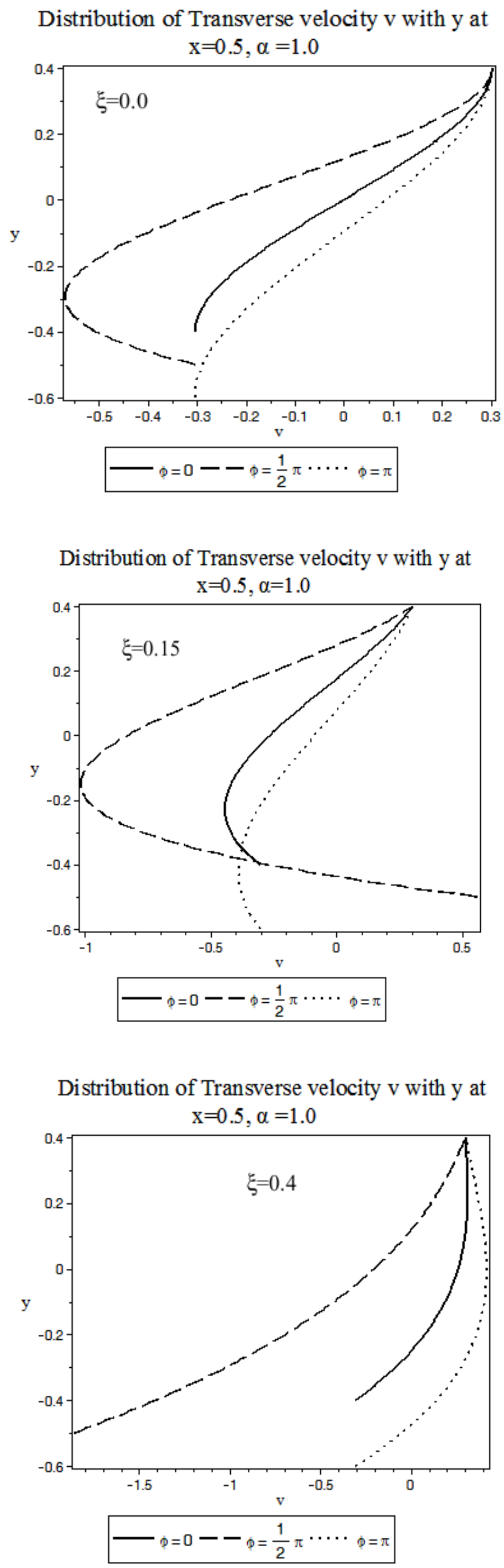
$$\begin{aligned}C_8(x)=&\left[\left(\xi+\frac{1}{2}\eta_2\right)\eta_1^5-\xi\eta_2\eta_1^4-\xi\eta_2^2\eta_1^3-\xi\eta_1^2\eta_2^3-\eta_2^4\left(\xi-\frac{1}{2}\eta_2\right)\eta_1+\xi\eta_2^5\right]A_5\\&+\left[\left(\frac{5}{3}\xi+\frac{5}{6}\eta_2\right)+\eta_1^4-\frac{5}{3}\xi\eta_1^3\eta_2-\frac{5}{3}\xi\eta_1^2\eta_2^2-\frac{5}{3}\eta_2^3\left(-\frac{1}{2}\eta_2+\xi\right)\eta_1+\frac{5}{3}\xi\eta_2^4\right]A_6\\&+\frac{5}{3}\left[\left(\xi+\frac{1}{2}\eta_2\right)\eta_1^3-\xi\eta_1^2\eta_2-\eta_2^2\left(-\frac{1}{2}\eta_2+\xi\right)\eta_1+\xi\eta_2^3\right]A_7\end{aligned}$$



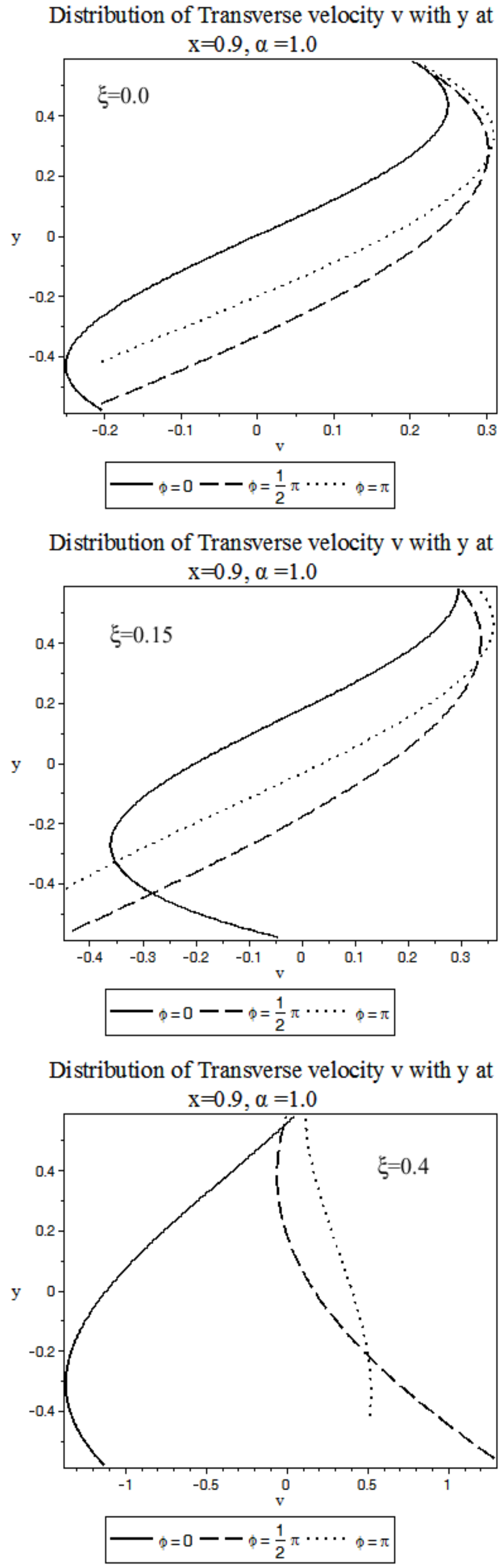




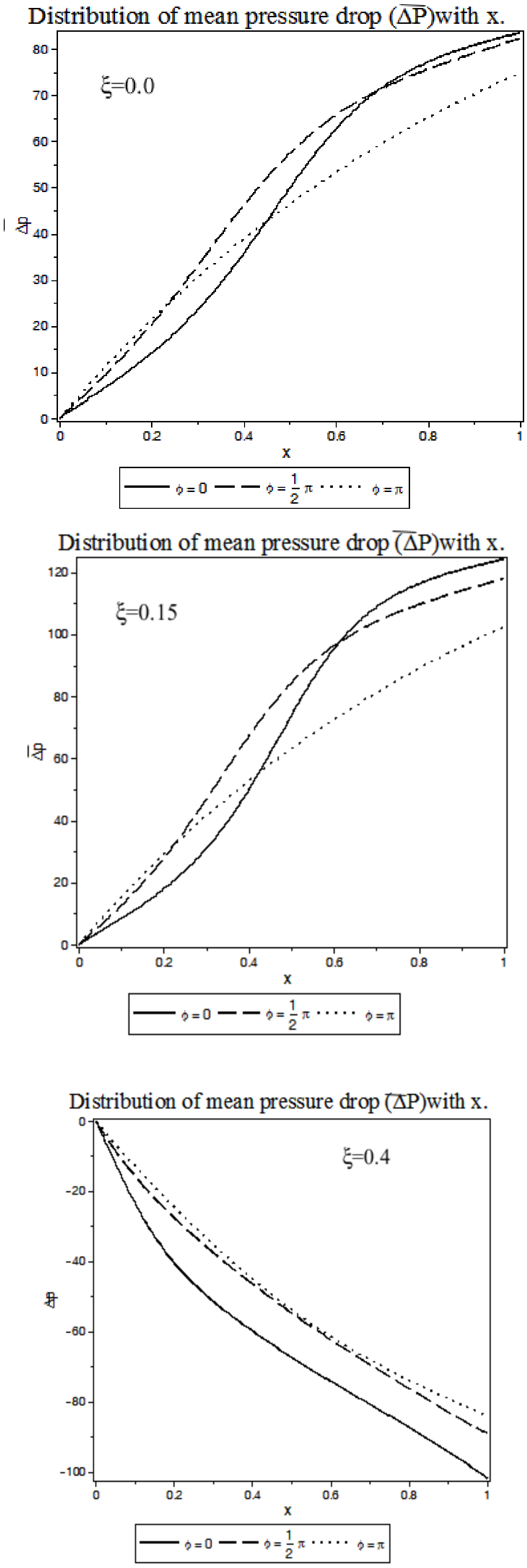
Figures 2(a)- 2(c) Distribution of Transverse velocity v with y at x=0.1, α=1.0



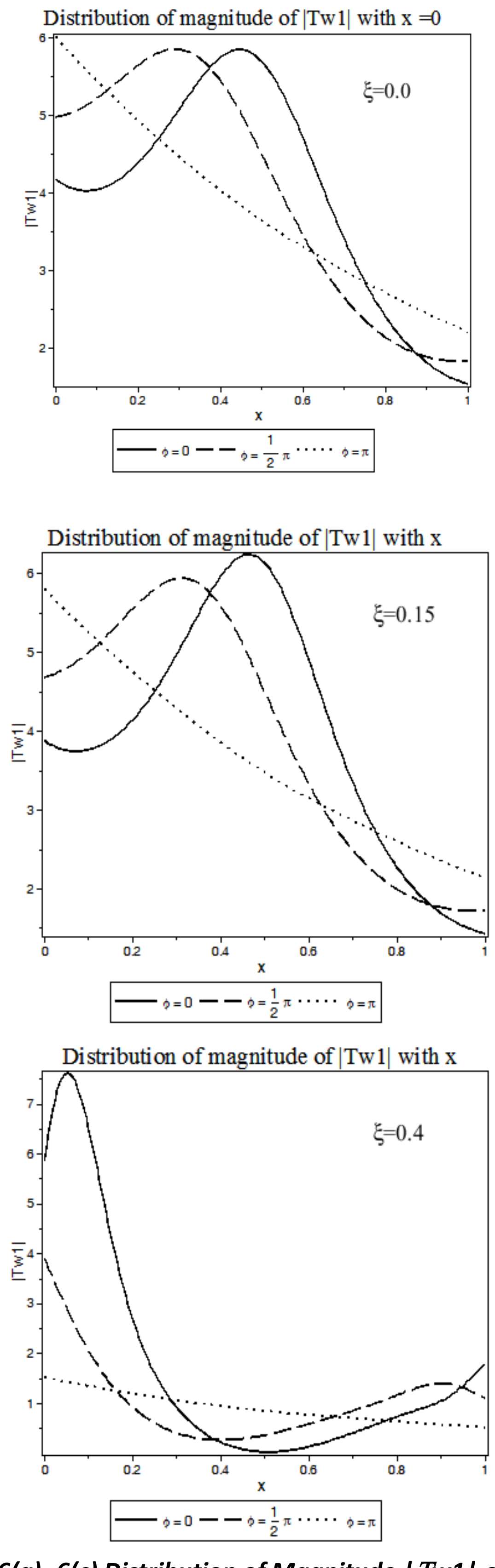
Figures 3(a)- 3(c) Distribution of Transverse velocity v with y at x=0.5, α=1.0



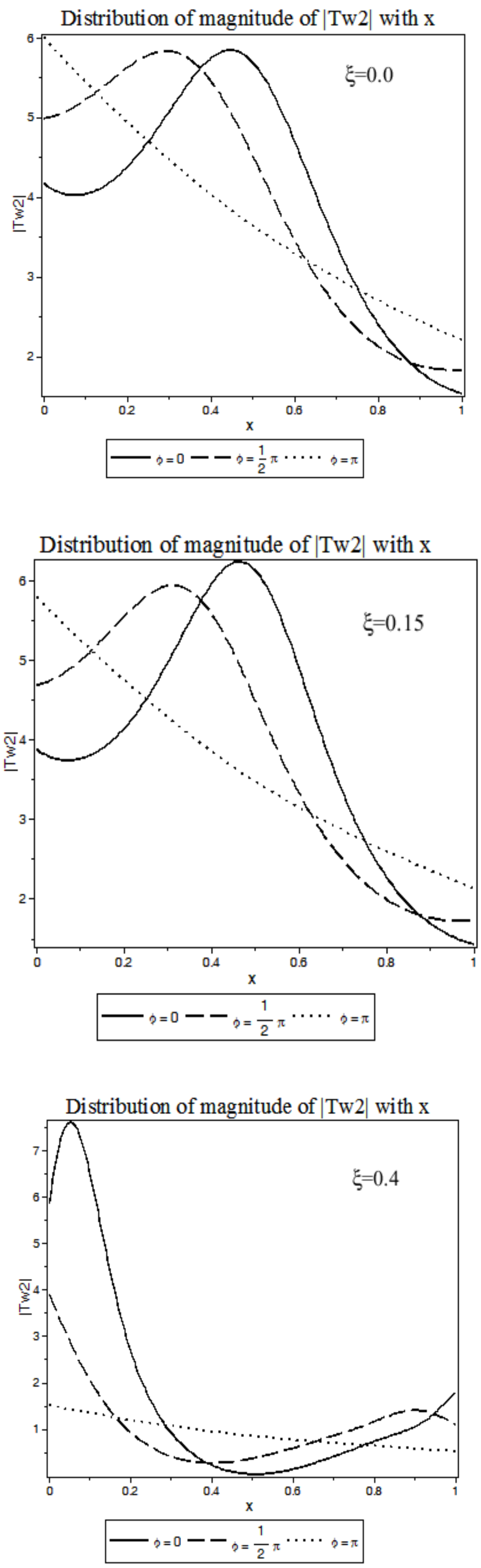
Figures 4(a)- 4(c) Distribution of Transverse velocity v with y at x=0.9, α=1.0



Figures 5(a)- 5(c) Distribution of Mean Pressure Drop with x



Figures 6(a)- 6(c) Distribution of Magnitude | Tw1| of with x



Figures 7(a)-7(c) Distribution of Magnitude | Tw2| of with x

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